

ac strain based thermodynamic criterion for vortex lattice in type-II superconductors

Peipei Lu,^{1,2,*} Mengju Yuan,^{1,*} Jing Zhang,¹ Qiang Gao,³ Shuang Liu,⁴ Yugang Zhang,¹ Shipeng Shen,³ Long Zhang,¹ Jun Lu³, Xiaoyuan Zhou,⁴ Mingquan He,¹ Aifeng Wang,¹ Yang Li,^{3,5} Wenshan Hong,³ Shiliang Li,³ Huiqian Luo,³ Xingjiang Zhou³, Xianhui Chen,⁶ Young Sun^{3,4,†} and Yisheng Chai^{1,4,‡}

¹Low Temperature Physics Laboratory, College of Physics, *Chongqing University*, Chongqing 401331, China

²College of Physics and Hebei Advanced Thin Films Laboratory, *Hebei Normal University*, Shijiazhuang 050024, Hebei, China

³Beijing National Laboratory for Condensed Matter Physics, *Institute of Physics, Chinese Academy of Sciences*, Beijing 100190, China

⁴Center of Quantum Materials and Devices, *Chongqing University*, Chongqing 401331, China

⁵University of Chinese Academy of Sciences, Beijing 100190, China

⁶Department of Physics, *University of Science and Technology of China*, Hefei, Anhui, China



(Received 1 July 2025; revised 1 May 2026; accepted 14 May 2026; published 9 June 2026)

We report a dynamic magnetostrictive effect in type-II superconductors (e.g., Nb, YBa₂Cu₃O_{7-x}, Bi₂Sr₂CaCu₂O_{8+δ} and Ba_{0.6}K_{0.4}Fe₂As₂) and any type-II superconductor is expected to present a similar response to this new technique. Measured via a composite magnetoelectric technique, an ac field excites an in-phase, nondissipative strain response scaling linearly with vortex density in the vortex lattice phase. In the vortex liquid phase, the signal acquires an out-of-phase component before vanishing in the normal state. We propose the ac strain susceptibility a thermodynamic criterion for the vortex lattice, capturing vortex collective modes inaccessible from static measurements.

DOI: [10.1103/gl5n-cc1n](https://doi.org/10.1103/gl5n-cc1n)

I. INTRODUCTION

Superconductors are defined by two hallmark properties: zero electrical resistivity and perfect diamagnetism, known as the Meissner effect [1,2]. These properties persist until superconductivity is quenched by a magnetic field exceeding a critical threshold H_c in type-I superconductors. By comparison, in type-II superconductors, the negative interfacial energy between superconducting and normal regions allows magnetic flux to penetrate the material above a lower critical field H_{c1} . This flux forms discrete vortices, each carrying a single flux quantum $\phi_0 = hc/2e$ (where h is the Planck constant and e is the electron charge). The quantum nature of these vortices facilitates a range of emergent phenomena, from their potential use as superconducting qubits to hosting Majorana zero modes in topological quantum computing [3,4]. These vortices typically arrange into regular two-dimensional lattices—often triangular or square in symmetry [5]. As temperature (T) or magnetic field increases, the rigid vortex lattice can melt into a dissipative vortex-liquid state above the irreversibility line (T_{irr} or H_{irr}), ultimately transitioning to the normal state at the upper critical field (H_{c2}). The static and dynamic properties of these phases play a central role in shaping the magnetic and transport responses of the material. For instance, hysteresis in magnetization loops $M(H)$ and the divergence between zero-field-cooled (ZFC) and field-cooled (FC) magnetization curves are classical manifestations of vortex pinning [6]. Similarly, the critical current density J_c , is

determined by the ability of the vortex lattice to remain static under an applied current [7], while, the Nernst effect provides a hallmark signature of vortex mobility in the vortex-liquid regime [8–10].

Complementary to these macroscopic probes, the thermodynamic strain channel provides a complementary probe of vortex phases via magnetostriction ($\lambda = \Delta L/L$, where L is the sample dimension). In type-II superconductors, static magnetostriction λ_{dc} arises from magnetoelastic coupling between the vortex system and the parent crystal lattice. It can be characterized using conventional strain gauges or capacitive dilatometers, often revealing butterfly-shaped hysteresis loops or discontinuous jumps due to thermomagnetic flux avalanches [11–15]. However, while such measurements only capture the quasistatic state, they do not easily isolate the collective excitation mode [16,17] or dissipation behavior within the vortex phases.

In contrast, the magnetic strain susceptibility, $(d\lambda/dH)$, opens a window onto the dynamical aspects of the vortex system. To formalize this, we consider the Gibbs free energy $G(H, \sigma, T)$, where σ is stress. The field derivative of strain, $(\partial\lambda/\partial H)_{\sigma, T}$, is a mixed second derivative of the free energy (a Maxwell relation) and thus a bona fide strain susceptibility linked to magnetization via $(\partial\lambda/\partial H)_{\sigma, T} = -(\partial M/\partial \sigma)_{H, T}$. Historically, $d\lambda/dH$ has been obtained by differentiating $\lambda(H)$ data from dc measurements, a method cannot probe dynamic and dissipative contributions [18]. Under a small ac drive, however, this susceptibility is complex:

$$(d\lambda/dH)_{ac} = d\lambda'/dH + id\lambda''/dH, \quad (1)$$

where the real part $d\lambda'/dH$ represents an elastic response, while the imaginary part $d\lambda''/dH$ quantifies dissipation.

*These authors contributed equally to this work.

†Contact author: youngsun@cqu.edu.cn

‡Contact author: yschai@cqu.edu.cn

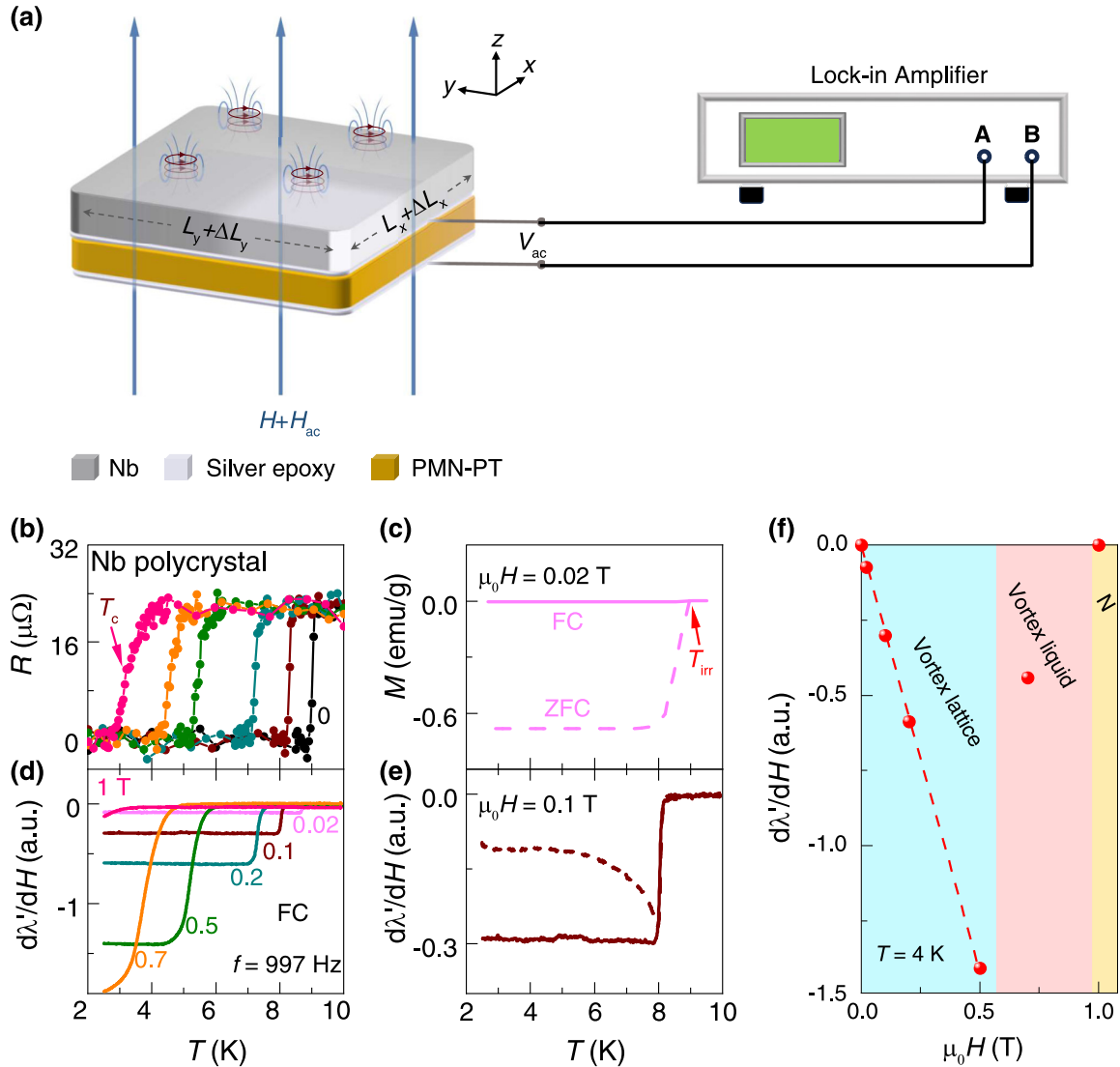


FIG. 1. (a) Schematic diagram of the Nb/PMN-PT composite configuration used for dynamic magnetostrictive measurements. T dependent (b), R under various H (c), magnetization in FC and ZFC processes under 0.02 T, (d), $d\lambda'/dH$ in FC process under selected H , and (e), $d\lambda'/dH$ in ZFC and FC processes under 0.1 T for Nb polycrystal. (f), H dependent $d\lambda'/dH$ at 4 K, extracted from (d).

Accessing this complex susceptibility therefore offers a practical strategy to probe the dynamic/dissipative properties of vortex phases.

It is only recently that this complex quantity has been measured with ultrahigh precision using a composite magnetoelectric (ME) technique. This method has been successfully applied to study the dynamics of quantized skyrmion lattices, other magnetically ordered systems and even quantum oscillation in topological semimetals [19–25]. The ME technique relies on a mechanically bonded heterostructure where length oscillations driven by a small ac field ($H_{ac} \approx 0.5 - 1$ Oe; $f \leq 10$ kHz) are converted into a measurable ac voltage (V_{ac}) by an attached piezoelectric layer [Fig. 1(a)]. A lock-in amplifier reads the voltage at high speeds (1–5 point/s), providing a significant advantage over conventional ac susceptibility. As the generated voltage scales with the complex transverse magnetostrictive coefficient ($V_{ac} \sim (d\lambda/dH)_{ac}$), we treat the measured signal as a direct proxy for the thermodynamic strain susceptibility.

In this article, we report the observation of a dynamic magnetostrictive effect in archetypal type-II superconductors, including Nb, $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ polycrystals and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ single crystals. We reveal a robust linear correlation between the amplitude of deformation and the vortex density in the lattice phase across all systems. Notably, we demonstrate that this dynamic effect is distinct from the static magnetostriction, evidenced by a stark contrast between their respective coefficients during flux avalanches and at the irreversibility line. These features establish the ac strain susceptibility as a robust thermodynamic criterion for identifying the vortex lattice state.

II. METHODS

Commercial polycrystalline Nb and $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ (YBCO) polycrystals samples were obtained from Alfa Aesar and the Central Iron & Steel Research Institute (China), respectively. Optimally doped and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

(BSCCO) and $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ (BKFA) single crystals were synthesized using the traveling solvent floating-zone method and the self-flux method, respectively [26–28]. Resistance was measured using a standard four-probe configuration. Magnetization data were collected using the vibrating sample magnetometer module of the physical property measurement system (PPMS) system. Ac magnetic susceptibility $\chi = \chi' - i\chi''$ was measured for Nb, YBCO, BSCCO, and BKFA samples using the ac measurement system (ACMS) option of the PPMS, with variable excitation frequencies. Dc magnetostriction is characterized with a commercial CuBi dilatometer (MultiField Tech.) with high precision capacitance bridge AH2700 (Andeen-Hagerling). Composite ME structures were prepared by bonding each type-II superconductor to a piezoelectric $0.7\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$ - 0.3PbTiO_3 (PMN-PT) [001]-cut single crystal (thickness = 0.2 mm) using silver epoxy (Epo-Tek H20E, Epoxy Technology Inc.). Prior to electrical measurements, the PMN-PT substrates were poled along the thickness direction using an electric field of 5.5 kV/cm for 1 h at room temperature. V_{ac} was measured using a lock-in amplifier (NF Corporation LI5645) integrated with a commercial sample stick (MultiField Tech.), as illustrated in Fig. 1(a). All the $d\lambda'/dH$ values were corrected by subtracting a small background signal from the normal state, which was either constant or linearly dependent on H_{dc} .

III. RESULTS AND DISCUSSION

A. Dynamics magnetostrictive effect in polycrystalline Nb

We begin with the investigation of the classical low- T_c superconductor Nb. The superconducting transition was characterized using conventional resistance (R) and dc magnetization measurements. As shown in Fig. 1(b), the transition temperature (T_c) is approximately 9.0 K at zero magnetic field, consistent with literature values [29,30]. T_c is progressively suppressed by the magnetic field, reaching ~ 3 K at 1 T. This is corroborated by $M(T)$ curves [see Supplemental Material (SM) for details] [31]. Under a field of 0.02 T, a pronounced separation between ZFC and FC magnetization curves indicates strong diamagnetism [superconducting volume fraction $\sim 68.8\%$, Fig. 1(c)]. The convergence of ZFC and FC curves at $T_{\text{irr}} = 8.8$ K marks the transition from the frozen vortex-lattice to the mobile vortex-liquid phase.

To investigate the dynamic magnetostrictive properties, a Nb/PMN-PT composite structure was fabricated. Figure 1(d) presents the temperature dependence of $(d\lambda/dH)_{\text{ac}}$ measured during FC processes. In the absence of a dc magnetic field, the signal remains negligible zero both below and above T_c , indicating no dynamic magnetostriction without vortex formation. However, when a finite magnetic field (up to 0.7 T) is applied, the real part $d\lambda'/dH$ exhibits a sharp drop at T_c , forming a pronounced negative plateau at lower temperatures. Simultaneously, the imaginary part $d\lambda''/dH$ displays a single dip centered around the transition region (see SM [31]). Both features are gradually suppressed with increasing field, vanishing completely near the upper critical field $H_{c2} \sim 0.8$ T [33].

The emergence of a nonzero $d\lambda''/dH$ near T_c is attributed to dissipation in the vortex-liquid phase (see SM [31]). To validate this, we measured the ac magnetic susceptibility $\chi = \chi' - i\chi''$ under $\mu_0 H_{\text{dc}} = 0.5$ T (see SM [31]). The temperature range of the $d\lambda''/dH$ dip coincides precisely with the peak in the magnetic loss signal χ'' . This supports the assignment of the imaginary strain response to dissipative vortex dynamics due to depinning. Accordingly, we, therefore, use the upper and lower bounds of the $d\lambda''/dH$ dip as operational estimates of T_c and T_{irr} , respectively, and compare them with resistance and magnetization criteria (see SM [31]). This criterion is further verified by comparing ZFC and FC $(d\lambda/dH)_{\text{ac}}$ curves [Fig. 1(e)]. At low temperatures, clear differences emerge due to flux trapping, but the curves converge precisely at T_{irr} , the onset of the dissipative dip (see SM [31]).

Detailed analysis of the $d\lambda'/dH$ plateau at 4 K reveals that its magnitude increases nearly linearly with magnetic field strength up to 0.7 T [Fig. 1(f)]. The finite $d\lambda'/dH$ in the vortex-lattice phase cannot arise solely from the applied background field H_{dc} , as the ZFC and FC results differ markedly. A more plausible interpretation is that the magnitude of $d\lambda'/dH$ directly reflects the vortex density. This hypothesis is supported by the ZFC data, which shows an increase in signal magnitude with rising temperature as thermal fluctuations facilitate the entry of vortices into the lattice.

To distinguish the dynamic response from static deformation, we performed field-dependent measurements on Nb at 2.5 K (Fig. 2). The magnetization $M(H)$ exhibits strong diamagnetism and deviates from linearity at $H_{c1} \sim 0.13$ T. Above H_{c1} , the loops display pronounced hysteresis and discrete magnetization jumps—flux avalanches—large number of vortices rush in or out of the sample in a very short time [see inset of Fig. 2(a)]. These avalanches persist up to 8.5 K (see SM [31]). Above H_{irr} , the hysteresis vanishes. In the ac magnetostrictive measurements [Fig. 2(b)], the real part $d\lambda'/dH$ does not exhibit a clear anomaly at H_{c1} . Instead, it tracks the vortex density: during vortex avalanches, $d\lambda'/dH$ exhibits steplike increases and decreases, while the imaginary part $d\lambda''/dH$ remains zero. The amplitude of $d\lambda'/dH$ peaks at H_{irr} , where the vortex density in lattice phase is maximal. At higher fields ($H > H_{\text{irr}}$), where $M(H)$ is nearly zero, $d\lambda'/dH$ rapidly decreases, vanishing near H_{c2} . In the intermediate range between H_{irr} and H_{c2} , the $d\lambda''/dH$ signal exhibits a single dip, consistent with the dissipative vortex-liquid phase observed in temperature sweeps.

The dc transverse magnetostriction λ_{dc} [Fig. 2(c)] displays a butterfly-shaped loop characteristic of type-II superconductors [11,13–15], with abrupt jumps matching the vortex avalanches. However, the derivative of this static strain, $d\lambda_{\text{dc}}/dH$ [Fig. 2(d)], reveals a fundamental difference. The $d\lambda_{\text{dc}}/dH$ are dominated by sharp, singular peaks during avalanches and vanish entirely at H_{irr} . In contrast, the dynamic coefficient $d\lambda'/dH$ shows monotonic steps during avalanches and reaches its maximum magnitude at H_{irr} . This pronounced discrepancy proves that the two measurements probe distinct physical phenomena. The dc coefficient reflects macroscopic, quasistatic flux redistribution (which stops at H_{irr}), whereas the ac coefficient probes the intrinsic, collective elasticity of the pinned vortex lattice itself, which is maximally ensembles just before melting.

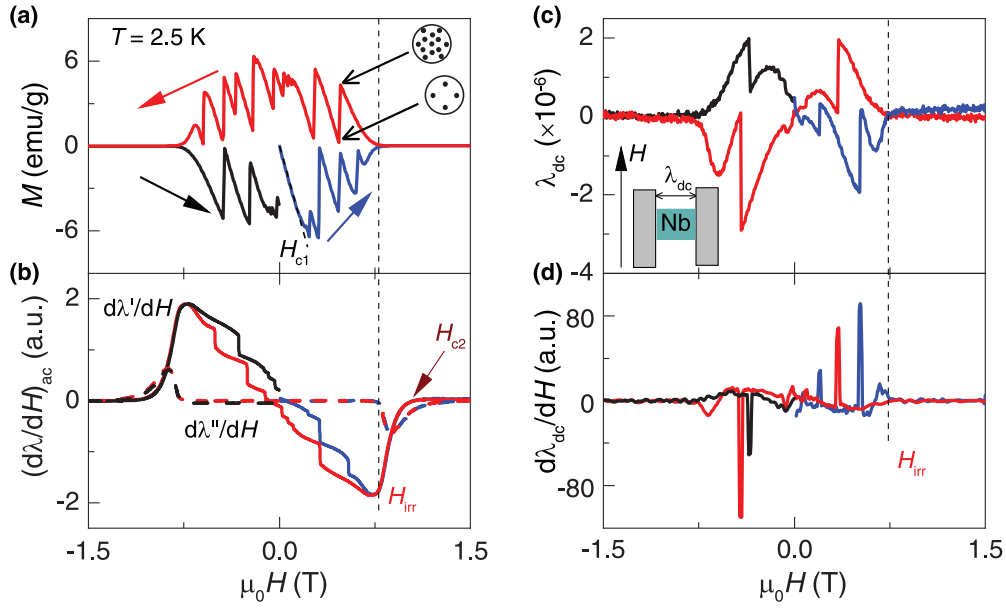


FIG. 2. H dependent (a), M , (b), $d\lambda'/dH$ and $d\lambda''/dH$, (c), transverse dc magnetostiction λ_{dc} , and (d), $d\lambda_{dc}/dH$ at 2.5 K for Nb polycrystal. The inset of (a) shows a schematic of a vortex avalanche event. The inset of (c) shows the measurement configuration for λ_{dc} . All measurements were performed after ZFC to 2.5 K.

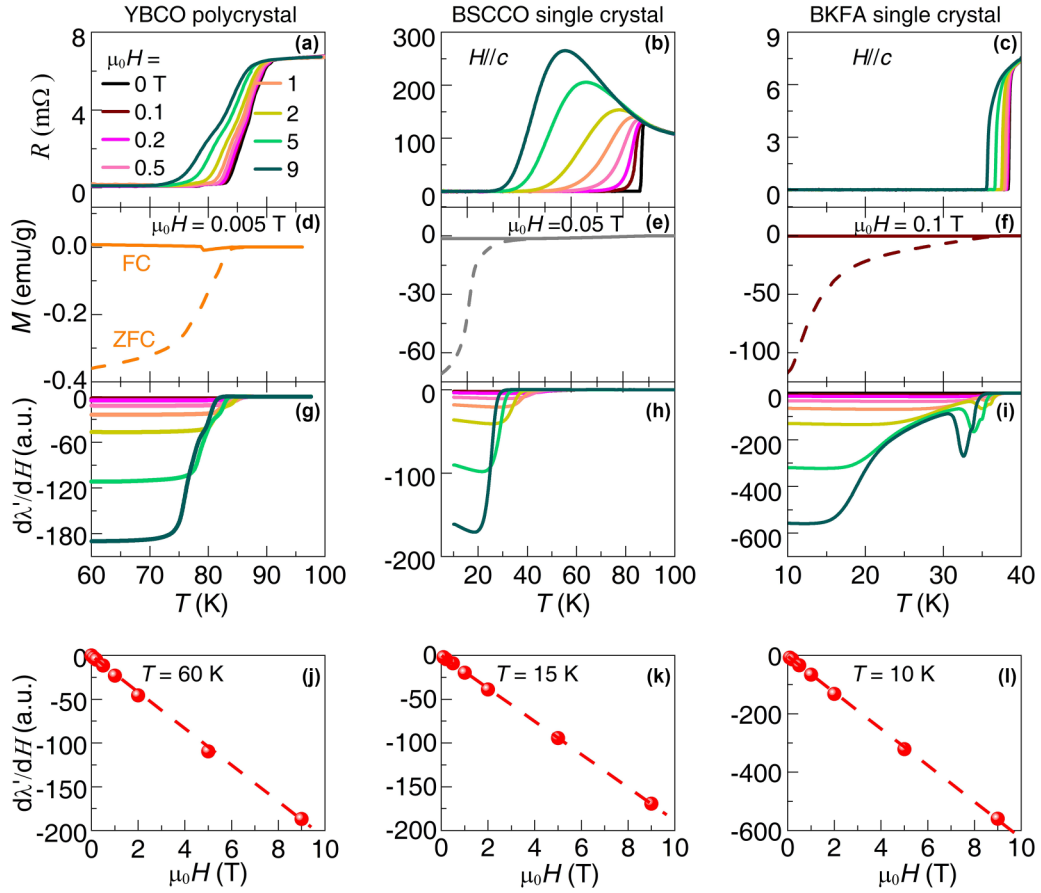


FIG. 3. (a)–(c), $R(T)$ in FC under selected H ; (d)–(f), $M(T)$ measured during FC and ZFC under magnetic fields of (d), 0.005 T (YBCO); (e), 0.05 T (BSCCO); (f), 0.1 T (BKFA); (g)–(i), $d\lambda'/dH(T)$ in the FC process under selected H ; (j)–(l), H dependent $d\lambda'/dH$ at selected T from data in (g) (YBCO), (h) (BSCCO), and (i) (BKFA).

B. Dynamics magnetostrictive effect in other high- T_c superconductors

To test the generality of this phenomenon, we extended our study to three additional archetypal superconductors: YBCO (Cu-based polycrystal), BSCCO (Cu-based single crystal), and BKFA (Fe-based single crystal). Resistance measurements yield transition temperatures of $T_c = 86.4$, 87.2 , and 38.5 K, respectively [Figs. 3(a)–3(c)]. The $R(T)$ curve for YBCO displays two steps, hinting at a vortex-slush phase [34]. Magnetization measurements confirm the type-II nature and the boundaries of the lattice phase [Figs. 3(d)–3(f)]. Composite ME structures were fabricated for each material. The temperature dependence of $d\lambda'/dH$ [Figs. 3(g)–3(i)] and $d\lambda''/dH$ (see SM [31]) was measured under selected dc fields. In all cases, $d\lambda''/dH$ reveals one or more dips, signaling the presence of dissipative phases. These dips coincide with peaks in the ac magnetic susceptibility χ'' , consistent with the observations in Nb (see SM [31]). Below T_{irr} , where $d\lambda''/dH$ vanishes, the real part $d\lambda'/dH$ exhibits a pronounced negative plateau (YBCO, BKFA) or a weakly temperature-dependent behavior (BSCCO). Specific material features are clearly resolved by the technique. In YBCO, an additional low-temperature peak in χ'' appears consistent with vortex shaking or depinning induced by the strong ac field. In BSCCO, the weak temperature dependence of $d\lambda'/dH$ reflects the weak pinning force, allowing thermal fluctuations to increase vortex penetration at higher temperatures. In BKFA, multiple dips in $d\lambda''/dH$ near T_{irr} reflect complex vortex dynamics, likely the coexistence of elastic and plastic vortex segments [35,36]. Despite these specific differences, the general phenomenology—a dissipative dip at the liquid phase and an elastic response in the lattice phase—is universal.

Furthermore, field-dependent measurements [Figs. 3(j)–3(l)] reconfirm the linear relationship between $d\lambda'/dH$ and vortex density. Both BSCCO and BKFA exhibit an almost linear increase of the $d\lambda'/dH$ amplitude with applied field, accompanied by minimal hysteresis. YBCO displays pronounced hysteresis consistent with stronger pinning in the polycrystalline form. Crucially, these behaviors differ fundamentally from the butterfly-shaped $d\lambda_{dc}/dH$ expected from static magnetostriction [11,13–15], reinforcing the distinct origin of the dynamic signal. The systematic observation of this effect across four distinct superconductors suggests a common vortex-related origin. We model this as a collective excitation of the vortex lattice.

We expect that a small variation in the external magnetic field δH induces two superimposed responses in crystal lattice (Fig. 4). First, a change in vortex density leads to a quasistatic deformation $\delta\lambda_{dc}$ via the interaction between vortices and pinning centers [2]. Second, rather than triggering depinning, each vortex line oscillates about its equilibrium position. Owing to boundary constraints (sample length L) and the dense packing of the lattice, a standing-wave-like collective mode with frequency ω_0 is excited. Prior microwave experiments place an upper limit on this mode of $\omega_0/2\pi \sim 10$ GHz in the absence of depinning [37]. Consequently, the measured ac magnetostrictive coefficient can be decomposed into two terms:

$$(d\lambda/dH)_{ac} = \delta\lambda_{dc}/H_{ac} + \delta\lambda_{ac}/H_{ac}. \quad (2)$$

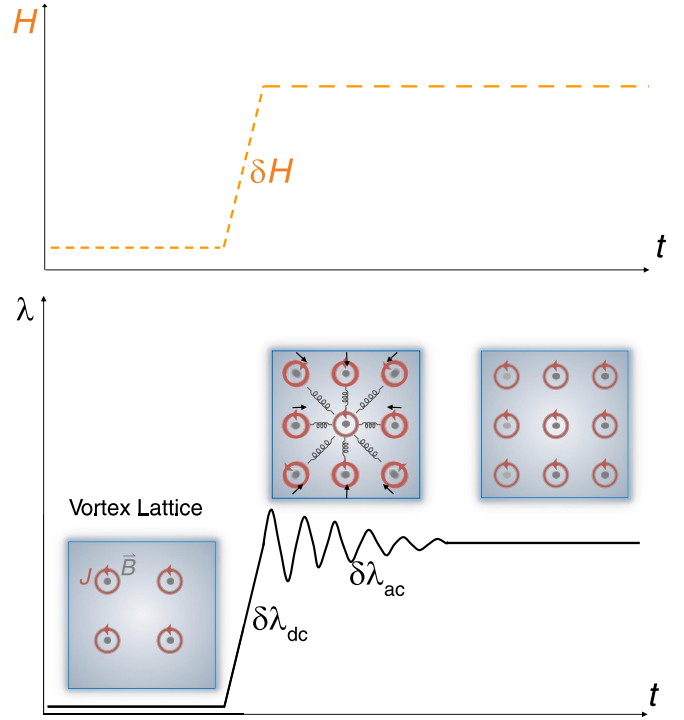


FIG. 4. A small variation of the external magnetic field δH excites collective oscillations of the vortex lattice without depinning, generating a dynamic strain response $\delta\lambda_{ac}$ that is superimposed on the static deformation $\delta\lambda_{dc}$ arising from the change in vortex density.

The first term reflects the static density change. The second, dynamic term can be approximated as

$$\delta\lambda_{ac}/H_{ac} \approx \frac{ng}{\omega_0^2}, \quad (3)$$

where n is the total number of vortices and g is the effective acceleration factor per vortex line (see SM for derivation [31]). For a typical sample area of 1 mm^2 under 1 T , the vortex number n reaches 10^9 . With ω_0 in the GHz range, the term ng/ω_0^2 would yield a measurable elastic response. This model successfully accounts for our key observations: (1) the large magnitude of the dynamic signal compared to the static derivative; (2) the linear scaling with vortex density ($n \sim \mu_0 H_{dc}$); and (3) the purely elastic nature (zero phase lag) within the lattice phase where the drive frequency ($< 10 \text{ kHz}$) is far below the resonant frequency ω_0 . In the vortex-liquid phase, vortex depinning, introducing dissipation and a finite imaginary component $d\lambda''/dH$. In the normal state ($n = 0$), the signal vanishes. This theoretical framework explains why the dynamic strain susceptibility provides information of the lattice complementary to static measurements.

From a practical standpoint, the measurement is rapid, robust, and requires only simple instrumentation, providing an accessible route to map vortex phase diagrams and quantify magnetoelastic coupling. To demonstrate its broader capability, we also applied this approach to the type-II superconductor $\text{EuFe}(\text{As}_{1-x}\text{P}_x)_2$, where superconductivity coexists with ferromagnetism. In this technique, two phases provide signals in opposite signs. For samples with $T_c < T_{FM}$ (T_{FM} is ferromagnetic transition temperature), the transition to a

ferromagnetic multidomain state suppresses H_{c2} and H_{irr} . Conversely, for samples with $T_c > T_{FM}$, strong short-range spin correlations and phase boundaries within a multiphase coexistence regime near a triple point leads to pronounced H_{c2} suppression [38].

IV. CONCLUSIONS

In conclusion, we have established the dynamic magnetostrictive effect as a strain-based thermodynamic criterion for the vortex lattice state, demonstrating a linear scaling with vortex density in type-II superconductors. Unlike static magnetostriction, the ac strain susceptibility isolates the collective elasticity of the pinned lattice from the dissipative liquid. This technique provides a rapid, sensitive probe of vortex matter, offering a powerful complement to transport and magnetization measurements for mapping phase diagrams of type-II superconductors.

ACKNOWLEDGMENTS

X.Y.Z. was financially supported from the National Science Fund for Distinguished Young Scholars (Grant No. 52125103). This work was supported by the National Natural Science Foundation of China (Grants No. 12227806, No.

12374081, No. 11674347, No. 11974065, No. 51725104, No. 11774399, No. 11474330, No. 52101221, No. U21A201910, No. 12574165, and No. 12274444), Fundamental Research Funds for the Central Universities (Project No. 2024IAIS-ZX002), the National Key Research and Development Program of China (Grants No. 2023YFA1406100 and No. 2025YFE0202100), the Central Guidance on Local Science and Technology Development Fund of Hebei Province (Grant No. 246Z7611G), Science Research Project of Hebei Education Department (Grant No. BJ2025091) and the Open Research Fund of the Pulsed High Magnetic Field Facility (Grant No. WHMFC2024007), Huazhong University of Science and Technology. Y.S.C. is grateful for the support from Beijing National Laboratory for Condensed Matter Physics. We thank G. W. Wang and Y. Liu at Analytical and Testing Center of Chongqing University for their assistance. We thank H. Y. Guo for his assistance in drawing the schematics.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request.

-
- [1] H. K. Onnes, Further experiments with liquid helium: The disappearance of the resistance of mercury, *Commun. Phys. Lab. Univ. Leiden* **122**, 13 (1911).
 - [2] W. Meissner and R. Ochsenfeld, Ein neuer effekt bei eintritt der supraleitfähigkeit, *Naturwissenschaften* **21**, 787 (1933).
 - [3] S. Mühlbauer, C. Pfleiderer, P. Böni, M. Laver, E. M. Forgan, D. Fort, U. Keiderling, and G. Behr, Morphology of the superconducting vortex lattice in ultrapure niobium, *Phys. Rev. Lett.* **102**, 136408 (2009).
 - [4] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Vortices in high-temperature superconductors, *Rev. Mod. Phys.* **66**, 1125 (1994).
 - [5] J. Y. Ge, J. Gutierrez, A. Lyashchenko, V. Filipov, J. Li, and V. V. Moshchalkov, Direct visualization of vortex pattern transition in ZrB_{12} with Ginzburg-Landau parameter close to the dual point, *Phys. Rev. B* **90**, 184511 (2014).
 - [6] A. Galluzzi, A. Nigro, R. Fittipaldi, A. Guarino, S. Pace, and M. Polichetti, DC magnetic characterization and pinning analysis on $\text{Nd}_{1.85}\text{Ce}_{0.15}\text{CuO}_4$ cuprate superconductor, *J. Magn. Magn. Mater.* **475**, 125 (2019).
 - [7] J. A. Fendrich, W. K. Kwok, J. Giapintzakis, C. J. van der Beek, V. M. Vinokur, S. Fleshier, U. Welp, H. K. Viswanathan, and G. W. Crabtree, Vortex liquid state in an electron irradiated untwinned $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ Crystal, *Phys. Rev. Lett.* **74**, 1210 (1995).
 - [8] Z. A. Xu, N. P. Ong, Y. Wang, T. Kakeshita, and S. Uchida, Vortex-like excitations and the onset of superconducting phase fluctuation in underdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$, *Nature (London)* **406**, 486 (2000).
 - [9] Y. Wang, L. Li, and N. P. Ong, Nernst effect in high- T_c superconductors, *Phys. Rev. B* **73**, 024510 (2006).
 - [10] A. Pourret, H. Aubin, J. Lesueur, C. A. Marrache-Kikuchi, L. L. Bergé, and K. B. Dumoulin, Observation of the Nernst signal generated by fluctuating Cooper pairs, *Nat. Phys.* **2**, 683 (2006).
 - [11] A. Nabialek, H. Szymczak, and V. V. Chabanenko, Pinning of the vortex system and magnetostriction of superconductors, *J. Low Temp. Phys.* **139**, 309 (2005).
 - [12] X. Yang, Z. Y. Du, H. Lin, D. L. Fang, H. Yang, X. Y. Zhu, and H. H. Wen, Vortex lattice and vortex bound states in CsFe_2As_2 investigated by scanning tunneling microscopy/spectroscopy, *Phys. Rev. B* **98**, 024505 (2018).
 - [13] A. Nabialek, H. Szymczak, V. A. Sirenko, and A. I. D'yachenko, Influence of the real shape of a sample on the pinning induced magnetostriction, *J. Appl. Phys.* **84**, 3770 (1998).
 - [14] M. J. Zhang, J. T. Wu, K. Shi, L. S. Ling, W. Tong, C. Y. Xi, L. Pi, J. Wosnitza, H. Q. Luo, and Z. S. Wang, Huge magnetostriction in superconducting single-crystalline $\text{BaFe}_{1.908}\text{Ni}_{0.092}\text{As}_2$, *Appl. Phys. Lett.* **123**, 072602 (2023).
 - [15] H. Ikuta, N. Hirota, Y. Nakayama, K. Kishio, and K. Kitazawa, Giant magnetostriction in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystal in the superconducting state and its mechanism, *Phys. Rev. Lett.* **70**, 2166 (1993).
 - [16] P. G. De Gennes and J. A. Matricon, Collective modes of vortex lines in superconductors of the second kind, *Rev. Mod. Phys.* **36**, 45 (1964).
 - [17] O. V. Dobrovolskiy, D. Y. Vodolazov, F. Porrati, R. Sachser, V. M. Bevz, M. Y. Mikhailov, A. V. Chumak, and M. Huth, Ultrafast vortex motion in a direct-write Nb-C superconductor, *Nat. Commun.* **11**, 3291 (2020).
 - [18] S. Spachmann, S. Selzer, B. Büchner, S. Aswartham, and R. Klingeler, Strong uniaxial pressure dependencies evidencing

- spin-lattice coupling and spin fluctuations in $\text{Cr}_2\text{Ge}_2\text{Te}_6$, *Phys. Rev. B* **107**, 184421 (2023).
- [19] Y. S. Chai, P. P. Lu, H. F. Du, J. X. Shen, Y. N. N. Ma, K. Zhai, L. Wang, Y. G. Shi, H. Li, W. H. Wang, and Y. Sun, Probe of skyrmion phases and dynamics in MnSi via the magnetoelectric effect in a composite configuration, *Phys. Rev. B* **104**, L100413 (2021).
- [20] P. P. Lu, H. F. Du, L. Wang, H. Li, W. H. Wang, Y. G. Shi, X. L. Wu, Y. Sun, and Y. S. Chai, Comparison of skyrmion phases between poly- and single-crystal MnSi by composite magnetoelectric method, *Appl. Phys. Lett.* **120**, 182406 (2022).
- [21] H. Zeng, X. W. Zhao, G. Yu, X. H. Luo, S. C. Ma, C. C. Chen, Z. J. Mo, Y. G. Zhang, Y. S. Chai, J. Shen, and Z. C. Zhong, Magnetic and transport properties of chiral magnet $\text{Co}_7\text{Zn}_8\text{Mn}_5$, *J. Magn. Magn. Mater.* **560**, 169631 (2022).
- [22] Y. G. Zhang, Z. F. Li, J. Zhang, N. Cao, L. Zhang, Y. H. Li, S. Liu, X. Y. Zhou, Y. Sun, W. H. Wang, and Y. S. Chai, Observation of enhanced ferromagnetic spin-spin correlations at a triple point in quasi-two-dimensional magnets, *Phys. Rev. B* **107**, 134417 (2023).
- [23] L. Zhang, T. Y. Wang, Y. G. Zhang, S. Liu, Y. P. Sun, X. Y. Zhou, Y. Sun, M. Q. He, A. F. Wang, X. Luo, and Y. S. Chai, Comprehensive investigation of quantum oscillations in semimetal using an ac composite magnetoelectric technique with ultrahigh sensitivity, *npj Quantum Mater.* **9**, 11 (2024).
- [24] L. Zhang, Z. Z. Jiang, Y. G. Zhang, J. Zhang, A. F. Wang, M. Q. He, Y. P. Sun, X. Luo, and Y. S. Chai, Magnetic phase diagram of Cr_7Te_3 revisited by ac magnetostrictive coefficient, *Appl. Phys. Lett.* **127**, 012413 (2025).
- [25] X. R. Mi, X. T. Li, L. Zhang, Y. C. Gu, A. F. Wang, Y. Li, Y. S. Chai, and M. Q. He, Precisely tracking critical spin fluctuations using the ac magnetostriction coefficient: A case study on the Kitaev spin liquid candidate $\text{Na}_3\text{Co}_2\text{SbO}_6$, *Phys. Rev. B* **111**, 014417 (2025).
- [26] J. F. He, W. T. Zhang, J. M. Bok, D. X. Mou, L. Zhao, Y. Y. Peng, S. L. He, G. D. Liu, X. L. Dong, J. Zhang, J. S. Wen, Z. J. Xu, G. D. Gu, X. Y. Wang, Q. J. Peng, Z. M. Wang, S. J. Zhang, F. Yang, C. T. Chen, Z. Y. Xu, H. Y. Choi, C. M. Varma, and X. J. Zhou, Coexistence of two sharp-mode couplings and their unusual momentum dependence in the superconducting state of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ revealed by laser-based angle-resolved photoemission, *Phys. Rev. Lett.* **111**, 107005 (2013).
- [27] H. Q. Luo, Z. S. Wang, H. Yang, P. Cheng, X. Y. Zhu, and H. H. Wen, Growth and characterization of $A_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ ($A = \text{Ba}$, Sr) single crystals with $x = 0-0.4$, *Supercond. Sci. Technol.* **21**, 125014 (2008).
- [28] Y. Li, D. S. Wu, Y. J. Shu, B. Liu, U. Stühr, G. C. Deng, A. P. J. Stampfl, L. Zhao, X. J. Zhou, S. L. Li, A. Pokhriyal, H. Ghosh, W. S. Hong, and H. Q. Luo, Neutron spin resonance near a Lifshitz transition in overdoped $\text{Ba}_{0.4}\text{K}_{0.6}\text{Fe}_2\text{As}_2$, *Chin. Phys. Lett.* **42**, 067405 (2025).
- [29] M. F. Schmidt, N. E. Israeloff, and A. M. Goldman, Applicability of high- T_c paradigms to magnetic relaxation and irreversibility in superconducting Nb, *Phys. Rev. B* **48**, 3404 (1993).
- [30] S. Salem-Sugui, M. M. Doria, A. D. Alvarenga, V. N. Vieira, P. F. Farinas, and J. P. Sinnecker, Average kinetic energy density of Cooper pairs above T_c in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, and Nb, *Phys. Rev. B* **76**, 132502 (2007).
- [31] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/gl5n-cc1n> for additional figures of Nb, YBCO polycrystal and BSCCO, BKFA single crystal, specific explanations for v , and subsequently ω_0 , specific formula derivation for the dynamic term, which includes Ref. [32].
- [32] E. H. Brandt, The vortex lattice in conventional and high- T_c superconductors, *Braz. J. Phys.* **32**, 675 (2002).
- [33] H. Takeya, Y. S. Sung, K. Hirata, and K. Togano, Superconducting properties of single-crystal Nb sphere formed by large-undercooling solidification process, *Physica C* **392-396**, 479 (2003).
- [34] T. Naito, H. Iwasaki, T. Nishizaki, S. Haraguchi, Y. Kawabata, K. Shibata, and N. Kobayashi, Vortex slush regime in the Josephson vortex phase diagram of 60-K $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ single crystals, *Phys. Rev. B* **68**, 224516 (2003).
- [35] W. Cheng, H. Lin, B. Shen, and H. H. Wen, Comparative study of vortex dynamics in $\text{CaKFe}_4\text{As}_4$ and $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ single crystals, *Sci. Bull.* **64**, 81 (2019).
- [36] K. Shi, W. S. Hong, Y. Li, M. J. Zhang, Y. Q. Han, Y. Zhao, J. T. Wu, Z. Wang, L. S. Ling, C. Y. Xi, L. Pi, H. Q. Luo, and Z. S. Wang, Doping dependence of resistivity, upper critical field and its anisotropy in overdoped $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ ($x = 0.6-1$) single crystals, *Chin. Phys. B* **35**, 017401 (2026).
- [37] R. Marcon, R. Fastampa, M. Giura, and E. Silva, Vortex-motion dissipation in high- T_c superconductors at microwave frequencies, *Phys. Rev. B* **43**, 2940 (1991).
- [38] M. J. Yuan, N. Zhou, R. X. Ti, L. Zhang, C. Y. Zhang, T. He, D. L. Ou, J. C. Gao, M. Q. He, A. F. Wang, J. Y. Ge, Y. Sun, and Y. S. Chai, Distinct suppression mechanisms of superconductivity by magnetic domains and spin fluctuations in $\text{EuFe}_2(\text{As}_{1-x}\text{P}_x)_2$ superconductors, [arXiv:2512.20364](https://arxiv.org/abs/2512.20364).